JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 100, NO. A1, PAGES 13–17, JANUARY 1, 1995

An empirical determination of the polytropic index for the free-streaming solar wind using Helios 1 data

T. L. Totten and J. W. Freeman
Rice University, Houston, Texas

S. Arya
Texas Southern University, Houston, Texas

Abstract. Observations of solar wind proton temperatures indicate that the solar wind is heated as it moves outward toward the orbit of Earth. This heating, which may be the result of electron heat conduction and perhaps MHD waves, has proven difficult to quantify and hence is often neglected in MHD models of the solar wind. An alternate approach to finding explicit heating terms for the MHD energy equation is to use a polytropic approximation. This paper discusses the properties of the polytropic approximation and its application to the solar wind plasma. By using data from the Helios 1 spacecraft, an empirical value for the polytropic index of the free-streaming solar wind is determined. Various corrections to the data are made to account for velocity gradients, nonuniformity in radial sampling, and stream interaction regions. The polytropic index, as derived from proton data, is found to be independent of speed state, within statistical error, and has an average value of 1.46. If magnetic pressure is included, the polytropic index has an average value of 1.58.

1. Introduction

Observations near the Sun and at Earth have indicated that the solar wind does not expand adiabatically in this region, implying that heating of the plasma occurs as it propagates through interplanetary space. An alternate approach to modeling a nonadiabatic fluid (or plasma) by using explicit heating terms in an energy equation is to utilize the polytropic approximation with a nonadiabatic exponent. Section 2 presents a discussion on the assumptions and meaning of the polytropic approximation as well as the implications for the solar wind. In section 3, data from the German spacecraft Helios 1 are used to determine an empirical value of the polytropic index of the protons in the free-streaming solar wind. Section 4 briefly addresses electrons. A discussion of results and conclusions are presented in section 5.

2. Physical Understanding of the Polytropic Equation

Chandrasekhar [1957] defines a quasi-static (reversible) process as one that occurs infinitely slowly so that at any given point in time, the system can be assumed to be in a state of thermal equilibrium. For a quasi-static process, the first law of thermodynamics can be written as follows:

$$dQ = dU + p\,dV. \tag{1}$$

Q represents the quantity of heat (per unit mass) added to or expelled from the system, U is the internal energy per unit mass of the system, p is pressure, and V is the specific volume. For an ideal gas, U is a function of only the temperature, T. Thus $dU = c_V\,dT$ where c_V is defined to be the specific heat of the gas at constant volume. By definition, a polytropic process is a quasi-static change of state in which the specific heat, $c = dQ/dT$, is held constant [*Chandrasekhar*, 1957]. Also for an ideal gas, $pV = RT$, where $R = (c_p - c_V)$ is the gas constant. Note that c_p is the specific heat at constant pressure. Applying these relations to equation (1), replacing the specific volume V with the mass density ρ, and integrating, one obtains the polytropic equation.

$$\frac{p}{\rho^\alpha} = \text{const.} \tag{2}$$

Here, α is called the polytropic index and can be written in terms of specific heats in the following manner:

$$\alpha = \frac{c_p - c}{c_v - c}. \tag{3}$$

Following the notation used by *Parker* [1963, 1965] and *Priest* [1982], the polytropic index is represented by the symbol α rather than γ (as is more commonly seen) to emphasize the fact that a polytropic expansion need not be adiabatic. The symbol γ is specifically reserved for the ratio of specific heats. $\gamma = c_p/c_V$ and is also related to the number of degrees of freedom, f, of the fluid; viz., $\gamma = (f+2)/f$ [*Farris et al.*, 1991]. Consequently, a polytropic expansion or compression is defined as a process in which the pressure and density vary according to equation (2).

All polytropic processes are, in theory, reversible. Furthermore, the power index may have any nonnegative value from zero to infinity [*Van Nostrand's Scientific Encyclopedia*, 1958]. An isobaric process is represented by $\alpha = 0$, and an isometric process has $\alpha = \infty$. For an isothermal expansion, $\alpha = 1$, which implies that the heat capacity, c, is infinite. Perhaps the most common polytropic approximation employed in models today is the adiabatic case. $\alpha = \gamma = c_p/c_V$, i.e., $c = 0$. Since a reversible, adiabatic process has constant entropy, this is also referred to as the isentropic case. If α is greater than 1, the temperature will decrease as the gas expands and increase as the gas is compressed. Also, if α is less than γ, heat must be supplied to the system in order for the fluid to expand [*Van Nostrand's Scientific Encyclopedia*, 1958].

Paper number 94JA02420.
0148-0227/95/94JA-02420$05.00

Observations of solar wind plasma have provided much information on some of the basic properties of this fluid as it propagates. Temperature measurements near the Sun and at Earth indicate that the solar wind cools as it expands, but does not cool rapidly enough to be considered an adiabatic expansion [*Neugebauer and Snyder*, 1966]. From the discussion presented above, observations of the solar wind plasma imply a polytropic index with a value greater than one but less than the adiabatic value. The solar wind plasma is regarded as having 3 degrees of freedom, implying $\gamma = 5/3$. Hence $1 < \alpha < 5/3$ for the solar wind.

This range of values for the polytropic index of the solar wind indicates that although the wind is being heated, the temperature falls as it expands. The reason for this is that the rate at which heat is added to the plasma is less than the rate at which work is done by the plasma in the process of expanding [*Van Nostrand's Scientific Encyclopedia*, 1958]. So how much heat is being added to the solar wind? The amount by which α is less than the adiabatic value of 5/3 can give an indication of the amount of heating that occurs [*Parker*, 1963; *Belcher*, 1971]. In fact, several researchers [*Parker*, 1965; *Belcher*, 1971; *Siscoe and Finley*, 1972; *Goldstein and Jokipii*, 1977; *Priest*, 1982; *Habbal*, 1985] have suggested that a nonadiabatic polytrope may roughly simulate the effects of heat conduction.

3. An Empirical Determination of the Polytropic Index for Solar Wind Protons

This section discusses the determination of an empirical value of the polytropic index for solar wind protons using data from the Helios 1 spacecraft. We assume the polytropic relationship applies to protons. The intent is not to neglect the effects of electrons, but rather to quantify the sum of all heating processes on protons through a polytropic index. We will discuss electrons in section 4. In what follows we will suppress the use of a p (for proton) subscript on all parameters.

Assuming the solar wind plasma proton parameters, namely temperature and number density, have power law relations with radial distance, a simple equation for the polytropic index is derived from the polytropic equation. Various corrections to the data are performed in order to address the effects of velocity gradients, nonuniformity in radial sampling, and heating due to stream-stream interactions. Using the corrected power indices for temperature and number density, we calculate the polytropic index for protons for seven solar wind speed states. We assume the protons are isotropic in the rest frame of the solar wind.

For an ideal, isotropic fluid, $p = nkT$ and $\rho = nm$ where n represents the number density (k represents Boltzmann's constant, and m is proton mass). Substituting these expressions into equation (2), we reduce the polytropic equation to the form below.

$$Tn^{(1-\alpha)} = \text{const.} \tag{4}$$

Taking the radial derivative of equation (4) and assuming α to be constant, we obtain the following expression (r is radial distance):

$$n\frac{dT}{dr} + (1-\alpha)\,T\frac{dn}{dr} = 0. \tag{5}$$

The assumption is made [*Schwenn*, 1983; *Freeman*, 1988] that the proton temperature and number density (T and n) are power law relations of the radial distance, r, in the range of the observations; viz.,

$$T \propto r^{-\delta} \qquad\qquad n \propto r^{-\beta}$$

where δ is the power index for the proton temperature and β is the power index for the proton number density. Substituting these forms for T and n into equation (5) and simplifying yields a simple equation for the polytropic index in terms of the power indices δ and β:

$$\alpha = 1 + \frac{\delta}{\beta} \tag{6}$$

One can easily verify that equation (6) produces the expected results for the limiting cases discussed in section 2. In particular, for adiabatic, spherically symmetric flow, $\delta = 4/3$ and $\beta = 2$, which yields the expected result of $\alpha = 5/3$.

The data for this calculation are for protons and are in the form of 1-hour averages. These data were collected by the Helios 1 spacecraft [See *Rosenbauer et al.*, 1977] and were obtained from the National Space Science Data Center, made available by R. Schwenn, F. Neubauer, and coworkers. Helios 1 has a highly eccentric orbit around the Sun with aphelion at 0.3 AU and perihelion at 1.0 AU. The data for this analysis span the time from launch in December 1974 through the year 1980. This 6-year period covers roughly one half of a solar cycle. The data set, taken as a whole, may be used to predict properties of the "free-streaming" solar wind because the large quantity of data causes an averaging effect. In other words, transient events that make up only a small portion of the data set will be "blended in" with the much more frequent events that represent the continuous, quiescent solar wind. With the assumption that the polytropic index is constant in the range from 0.3 to 1.0 AU, the polytropic index of the free-streaming solar wind can be determined by using the Helios 1 data.

The state of the solar wind is most easily characterized by speed [*Schwenn*, 1983]. Therefore the power indices for proton temperature and number density are determined by first sorting the data into 100-km/s speed bins and plotting against radial distance on a log-log plot. The highest speed range (velocities greater than 800 km/s) contains only 61 of the 41,076 data points used in this analysis and is found to be statistically unreliable throughout the calculations presented in this paper. The proton temperature [*Schwenn*, 1983; *Freeman*, 1988] and density, in each speed range, are both linear on a log-log plot against distance. A linear regression analysis is performed for proton temperature and number density in each speed range to obtain the power indices.

As mentioned earlier, three adjustments to the data for proton temperature and number density are made to account for the effects of nonzero speed gradients, nonuniformity in radial sampling, and heating due to stream-stream interactions. The first of these refers to the fact that the speed of the solar wind is not constant with radial distance but steadily increases [*Arya and Freeman*, 1991]. This property of the solar wind speed introduces a bias in sorting the data by speed state. Therefore it is prudent to normalize the speed data, using the velocity gradients determined by *Arya and Freeman* [1991], to some common radial distance, chosen here to be 1 AU, before sorting the temperature and number density data into speed bins.

The next correction deals with the fact that, due to the elliptical orbit of the spacecraft, the number of points in the data set is different for different radial distances. At aphelion, Helios 1 moves slowly and orthogonal to the radial direction; consequently, the highest density of data points is near 1 AU. The next highest density of points is at perihelion because the spacecraft is again moving orthogonal to the radial direction. In order to compensate for this effect, the data are divided into 0.1-AU bins, and the fraction of points in each bin (compared to the total number of points in the data set) is calculated. These fractions are used to weight each data point to eliminate any possible biasing by radial distance.

The final correction to the power indices concerns the possible heating effects of stream-stream interactions. Although the heating effects are predicted to be quite small [*Burlaga and Ogilvie*, 1973; *Lopez and Freeman*, 1986], this correction is performed to ensure that the values for the power indices are computed as accurately as possible. A number of computerized filters have been employed to remove data points thought to lie near stream interaction regions. These filters were designed to look at either 1-hour or 6-hour averages of solar wind bulk speed. If the data showed a sharp rise or fall in the flow speed, then the data at and near this sharp gradient were discarded. This approach is similar to that used by *Burlaga and Ogilvie* [1973], *Pizzo et al.* [1973], and *Lopez and Freeman* [1986]. However, none of these filters could discard the appropriate data satisfactorily. Consequently, the data in question were removed by hand based on plots of 1-hour averages of solar wind speed. Out of 43,696 hourly averages in the original data set, 2620 were removed by the hand-filtering process.

Table 1 shows the values for the power indices for proton temperature and number density, by speed range, after all the corrections to the data just described are performed. By using equation (6), the values for the polytropic index for each speed range are determined. (See column four of Table 1.) As mentioned earlier, the highest speed state (>800 km/s) has statistically unreliable results for all parameters due to the small number of data points in this range. Except for the highest speed range, the value for α is the same, within statistical error. The average value is found to be 1.46 ± 0.04. This result suggests that the heat flux for both the high- and low-speed streams is the same. Note that as the temperature index changes from one solar wind state to the next, the power index for density adjusts in such a manner as to keep α roughly constant.

The temperature indices in Table 1 differ from those published previously [*Schwenn*, 1983; *Freeman and Lopez*, 1985; *Lopez and Freeman*, 1986], particularly for the two lowest speed ranges, mainly because the earlier work did not take into account the effects of velocity gradients.

The solar wind magnetic pressure is typically slightly higher than the proton thermal pressure. It can be argued that the magnetic pressure should be included in the calculation of the polytropic index. Suppose that the magnetic pressure is included in the polytropic equation (2) so that p = proton pressure + magnetic pressure = $nkT + (B^2/2\mu_o)$.. The polytropic equation now has the form shown below (recall $\rho = nm$):

$$\frac{nkT + B^2/2\mu_o}{n^\alpha} = \text{const.} \tag{7}$$

As with number density and temperature, let the magnetic field magnitude, B, have a power law relation with radial distance, i.e.,

$$B \propto r^{-\lambda}.$$

Applying these forms for n, T, and B to equation (7), taking the radial derivative, and rearranging, we obtain the following expression:

$$\frac{\alpha\beta - 2\lambda}{\delta + \beta - \alpha\beta} = \frac{nkT}{B^2/2\mu_o} \equiv \text{plasma beta} \tag{8}$$

Recall that α is the polytropic index, and δ, β, and λ are the power indices for proton temperature, number density, and magnetic field magnitude, respectively. The values for δ and β have already been determined and are shown in Table 1. The Helios 1 magnetometer data [*Musmann et al.*, 1977] are used to calculate the power index for the magnetic field (λ) in the same manner, including all the same corrections, as the indices for the density and temperature are obtained from the plasma data. The results, by speed state, are shown in column two of Table 2. Column three of the same table shows the calculated values of the plasma beta at 1 AU, also determined by using the data from the Helios 1 spacecraft. These values are substituted into equation (8), and the new values for the polytropic index are determined for each speed range. The results are shown in Table 2. As in the case discussed previously, except for the highest speed range, the polytropic index is independent of speed state, within statistical error. The average value for α is 1.58 ± 0.06. Note that values for this polytropic index for several speed states are statistically equal to the adiabatic value of 5/3. However, the average value is slightly more than 1 standard deviation away from the adiabatic value. It should be pointed out that the data set used to calculate the quantities in Table 2 is not identical to the data set used to calculate the polytropic index considering particle pressure only, because the magnetometer and plasma experiments were not always operational during the same time periods.

An attempt has been made to determine the polytropic index directly from log-log plots of pressure versus number density.

Table 1. Temperature and Density Indices and the Polytropic Index for Several Solar Wind Speed Ranges

Speed Range, km/s	Temperature Index, δ	Density Index, β	Polytropic Index, α ($\alpha = 1 + \delta/\beta$)
<300	-0.93 ± 0.22	-2.09 ± 0.29	1.44 ± 0.12
300-400	-1.04 ± 0.12	-2.13 ± 0.11	1.49 ± 0.06
400-500	-0.99 ± 0.12	-2.03 ± 0.11	1.49 ± 0.07
500-600	-0.87 ± 0.13	-1.89 ± 0.13	1.46 ± 0.08
600-700	-0.81 ± 0.13	-1.93 ± 0.12	1.42 ± 0.07
700-800	-0.86 ± 0.28	-1.88 ± 0.23	1.46 ± 0.16
>800	0.93 ± 1.11	-1.95 ± 1.14	0.53 ± 0.63

The errors in the temperature and density indices represent 1 standard deviation in the linear regression analysis.

Table 2. Parameters Involving the Polytropic Index That Includes Magnetic Pressure

Speed Range, km/s	Magnetic Field Index, λ	Plasma Beta $nkT/(B^2/2\mu_o)$	Polytropic Index, α
<300	-1.74 ± 0.17	0.48 ± 0.20	1.59 ± 0.21
300-400	-1.65 ± 0.09	0.44 ± 0.06	1.53 ± 0.09
400-500	-1.72 ± 0.09	0.47 ± 0.07	1.63 ± 0.09
500-600	-1.71 ± 0.09	0.60 ± 0.09	1.68 ± 0.11
600-700	-1.57 ± 0.10	0.65 ± 0.10	1.55 ± 0.10
700-800	-1.45 ± 0.20	0.61 ± 0.18	1.51 ± 0.20
>800	-1.64 ± 0.28	0.16 ± 0.16	1.52 ± 0.86

After the temperature and density data are normalized to 1 AU, using the corrected power indices δ and β, the data points are found to scatter in all directions about a central point. The scatter is so great that an accurate determination of the polytropic index is impossible. It is not surprising that this method is less accurate than the process described earlier. *Zhu* [1990] discusses this approach as applied to calculating the polytropic index for the Earth's plasma sheet. He states that the scatter in log-log plots of pressure (or temperature) against number density is caused by variations of the specific entropy from one flux tube to another and that this scatter will affect the determination of the polytropic index. Consequently, the present process for calculating the polytropic index is presumed to be more accurate than the "log p versus log n" approach for the case of the solar wind.

As already stated, the average value for the polytropic index for solar wind protons is 1.46, neglecting the (unreliable) highest speed range. If the magnetic pressure is included, the average value for α is 1.58. As predicted, these values are less than the adiabatic value of 5/3 (1.66) but greater than the isothermal value of 1. This result conforms to the physical understanding of the polytropic equation and to in situ observations made of the solar wind. Specifically, heat is added to the plasma as it expands, yet the temperature declines with distance from the Sun. However, the empirical determination of α moves one step beyond what is already known by providing a quantitative, statistical representation of the heating that exists in the solar wind. Furthermore, the heating is found to be independent of solar wind state, within statistical accuracy.

Now that the statistical polytropic index has been calculated, only the "constant" in equation (2) remains unknown. Setting $\rho = mn$, where m is the proton mass, equation (2) can be written as follows:

$$\frac{p}{n^{\alpha}} = \text{const} \equiv C \qquad (9)$$

The constant C can be determined for protons at any specified point in the solar wind using the values for the pressure and number density corresponding to the specified point. Column two of Table 3 shows the values for C, by speed state, calculated at 0.3 AU. The values for the thermal pressure ($p = nkT$), number density, and polytropic index appropriate to each speed range are used to determine C. Because the polytropic index for the highest speed range (>800 km/s) is so unreliable, the constant for this range cannot be accurately determined. Column three of the same table shows a similar calculation at 1.0 AU. Comparing these two columns reveals that, for a given speed range, C does not vary significantly from 0.3 to 1.0 AU. On the other hand, the constant varies considerably from one speed state to another. Columns four and five of Table 3 display the values for C (at 0.3 and 1.0 AU, respectively) when magnetic pressure is included ($p = nkT + B^2/2\mu_0$). Again, C is roughly constant with radial distance, as expected, yet the values change significantly with speed range.

With the empirical determination of both constants (α and C) in equation (9), the polytropic relation for solar wind protons is completely defined. The polytropic equation may be used to close a set of equations describing the solar wind plasma, rather than employing a more complicated energy equation.

4. Solar Wind Electrons

A single-fluid thermodynamic description of the solar wind polytropic index is not complete without a specification of the electron contribution. Treatment of the electrons is more complicated because of the core and halo components and their anisotropy. They probably heat the ions significantly and also contribute to the total pressure. In the foregoing we have treated the proton gas alone. A description in terms of a polytropic index for the protons is still useful, since the polytropic index gives us a quantitative determination of the proton heating independent of the heating source.

The Helios spacecraft instrumentation included electron detectors; however, to the best of our knowledge, a comprehensive set of hourly averages of electron parameters versus radial distance has not been made available, and so a comparable analysis is not possible for electrons at this time.

Phillips and Gosling [1990, 1991], have developed a simple model of core electron transport, using the *Chew et al.* [1956] relations that neglects the electron heat flux but focuses on the roles of collisions and magnetic field geometry. The model is based on ISEE 3 data, an assumed temperature at the Sun, and spherical expansion. They find an adiabatic polytropic index for a collision-dominated electron gas and flatter radial gradients for the core temperature for smaller (more realistic) collision rates, but no specific values for α are given for the latter case.

5. Conclusions

We have obtained a statistical value for the polytropic index for solar wind protons, using data from the Helios 1 spacecraft. This empirical value is determined by using radial distance power law indices for the proton temperature and number density in a new relationship for α that depends only on these indices. The data are first corrected for stream-stream interactions and other effects and sorted by bulk speed. The values for the polytropic index for the several solar wind speed states are found to be independent of

Table 3. Values for the Constant C in Equation (9) by Speed State

Speed Range, km/s	0.3 AU $p = nkT$, 10^{-22} N m$^{3\alpha-2}$	1.0 AU $p = nkT$, 10^{-22} N m$^{3\alpha-2}$	0.3 AU $p = nkT, B^2/2\mu_0$, 10^{-22} N m$^{3\alpha-2}$	1.0 AU $p = nkT, B^2/2\mu_0$, 10^{-22} N m$^{3\alpha-2}$
>300	1.83 ± 1.30	1.83 ± 0.67	0.32 ± 0.45	0.50 ± 0.31
300-400	2.32 ± 0.76	2.32 ± 0.34	3.63 ± 1.76	3.77 ± 0.79
400-500	5.89 ± 1.92	5.89 ± 0.82	1.46 ± 0.75	2.06 ± 0.41
500-600	18.3 ± 6.15	18.3 ± 2.46	0.71 ± 0.42	1.78 ± 0.36
600-700	51.1 ± 16.1	51.1 ± 6.12	12.8 ± 6.13	18.7 ± 3.02
700-800	33.5 ± 22.2	33.5 ± 7.95	36.4 ± 31.5	38.9 ± 11.4

speed to within statistical errors. The average value of α is found to be 1.46 ± 0.04. If the effects of magnetic pressure are included, the average value becomes 1.58 ± 06. The constant C in the polytropic relation is also determined. This constant depends on speed state but is independent of radial distance.

Acknowledgments. This work has been supported by NASA JOVE grants NAG8-146 and NAG8-818 and Texas Advanced Technology Program grants 003604-01 and 003642-001. The Helios plasma and magnetic field data were provided to the National Space Science Data Center by R. Schwenn, F. Neubauer, and coworkers. We gratefully acknowledge useful discussions with Anthony Chan and Vytenis Vasyliunas.

The Editor thanks Z. Smith and another referee for their assistance in evaluating this paper.

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S. Arya, Texas Southern University, Houston, TX 77004. (e-mail: arya@spacvax.rice.edu)

J. W. Freeman and T. L. Totten, Department of Space Physics and Astronomy, Rice University, 6100 S. Main Street, P. O. Box 1892, Houston, TX 77251. (e-mail:: freeman@spacvax.rice.edu; totten@spacvax.rice.edu)

(Received March 15, 1994; revised September 13, 1994; accepted September 13, 1994.)