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Spatial evolution of the magnetic field spectral exponent in the solar wind: Helios and Ulysses comparison

E. Marsch
Max-Planck-Institut für Aeronomie, Katlenburg-Lindau, Germany

C.-Y. Tu
Max-Planck-Institut für Aeronomie, Katlenburg-Lindau, Germany
Department of Geophysics, Peking University, Beijing

Abstract. The spatial evolution of the spectral exponent of magnetic field fluctuations in the solar wind is investigated by a comparison of spectra and length functions obtained from Helios for in-ecliptic and from Ulysses for high-latitude observations. A similar radial evolution trend is found in both data sets for the B_z component of the magnetic field, which is least affected by compressive interactions. Yet the fluctuations seem to evolve more slowly in the polar flows as compared with the Alfvénic fluctuations found by the Helios experiments in coronal-hole-related flows near the Sun in the ecliptic plane.

Introduction

Recently, *Horbury et al.* [1995] estimated the spectral index of the interplanetary magnetic field fluctuations from the scaling exponents of the length function, which was constructed from Ulysses observations in the heliospheric latitudinal range from 41° to 51° and radial distance range from 4.2 to 3.7 AU. They concluded that at high heliographic latitudes the turbulent spectra in the polar solar wind flows did not strongly develop radially, even not out to 4 AU, but seemed to evolve more slowly than the spectra obtained near the ecliptic (for a review, see, e.g., *Tu and Marsch* [1995]).

The purpose of this short communication is to provide a more detailed comparison of the Helios and the Ulysses results. For this purpose we calculate the spectral index in two ways: from the exponent of the spectrum calculated by the fast Fourier transform (FFT) technique and from the length function. We will compare the results obtained in these two different ways.

Results derived from the length function are presented for two cases of Helios solar wind data measured near 0.3 and 1 AU, respectively. We then compare these results with the ones reached by *Horbury et al.* [1995]. We found a considerable evolution of the spectral index in the region extending from 0.3 to 4 AU and 41° to 51°.

Description of the Method

The method employed here has been described first in the solar wind context by *Burlaga and Klein* [1986] and then used by *Ruzmaikin et al.* [1993] and lately by *Horbury et al.* [1995]. The main idea is to interpret the observed time series of the magnetic field components, $B_j(t)$, in terms "fractal curves" and to analyze the scaling behavior of finite differences of the field with variable time lag τ. The function $B_j(t)$ may be viewed geometrically as a curve having structure on every scale. If the shape of the time series (curve) is statistically invariant under scale transformations, it is called self-affine, which means that each part of the curve can be considered a scale-reduced image of the entire curve. Its "length" (note that its units are in nanoteslas) is given by the formula:

$$L(\tau) = \sum_{k=1}^{N(\tau)} |B_j(t_k + \tau) - B_j(t_k)| \qquad (1)$$

where $N(\tau)$ is the number of points used to calculate the average. The length is a function of τ and, if statistically self-similar, has the scaling properties

$$L(\tau) \sim \tau^l \qquad (2)$$

with the scaling exponent l. The length of a curve is closely related with the magnetic field structure function, which has been analyzed in detail with Helios data by *Marsch and Liu* [1993]. Recently, *Horbury et al.* [1994] also employed a detailed structure function analysis. The variance of the component B_j is given by the second-order structure function and may scale as

$$(\delta B_j)^2 = \langle [B_j(t+\tau) - B_j(t)]^2 \rangle \sim \tau^{s(2)} \qquad (3)$$

with the exponent $s(2)$ of the structure function, which for statistical self-similarity is connected with the exponent α of the power spectrum through the relation $f\mathcal{P}(f) \sim (\delta B_j)^2$, which yields with $\mathcal{P}(f) \sim f^{-\alpha}$ the relation

$$s(2) = \alpha - 1 \qquad (4)$$

Since $N(\tau) = T/\tau$, where the length of the considered time series is T, we have with (3) used in (1) the scaling relation

$$L(\tau) \sim N(\tau)\delta B_j \sim \tau^{s(2)/2-1} \qquad (5)$$

Therefore one obtains from (4) and the above scaling relations the exponent

$$\alpha = 2l + 3 \qquad (6)$$

This shows how the exponent of the length function and the power spectrum are related to each other. Spectral exponents are usually determined from the power spectrum obtained by

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Fourier analysis. The fractal method is much simpler than the Fourier transformation and the scaling exponent l in (2) decribes directly a property of the magnetic field but not of its variance. To find l, one only needs to sum differences in the time series and to take the logarithm. For further discussion and the literature we refer to the papers by *Burlaga and Klein* [1986] and *Ruzmaikin et al.* [1993].

We would like to mention that in the evaluation of (1) we did not use all the data of the original time series but decimated the data, since only a subset of the data at the discrete times t_k, with the index k running from unity to $N(\tau)$ and the time differences obeying $t_{k+1} - t_k = \tau$, is really required in (1). Only if we use the same method to calculate the variance in (3) can we obtain the relation between the exponents in (6). Especially for flat spectra with $\alpha \leq 1$, this procedure ensures that the relation $f\mathcal{P}(f) \sim (\delta B_j)^2$ can still be used. However, considering aliasing effects, we must keep in mind that high-frequency fluctuations still influence $(\delta B_j)^2$, so that the above relation and (6) derived with its help cannot hold exactly even if we decimate the data.

Results of the Data Analysis

For our analysis we are going to use only the out-of-ecliptic magnetic field component B_z, which is largely Alfvénic in nature and less affected by compressive effects acting on the in-ecliptic components of the flow velocity and the magnetic field. The data analyzed here were obtained by the magnetometer onboard the Helios 2 spacecraft in 1976, in the first case considered during the days 104–109 and at solar distances near 0.3 AU, and in the second case during the days 21–26 at distances near 1 AU. For the first period the averaged proton radial speed is 695 km/s, and for the second period the averaged speed is 619 km/s. The data gaps are less than 8% of the whole set in both cases.

First, we make a comparison between the two indices derived from the length function and power spectrum that was calculated by the FFT technique. Figure 1a shows the index α calculated from the length function. The solid line shows the observations made near 0.3 AU, while the dashed line is for the 1 AU case. In the solid line of Figure 1a we see a plateau in the frequency range between 10^{-3} and 10^{-4} Hz, while the dashed line indicates a continuous increase of α with decreasing frequency. Both curves tend, though with a rugged shape, to flatten somewhat near their low-frequency ends.

For a comparison we show in Figure 1b the power spectra, which are calculated applying the FFT technique to the same data used for producing Figure 1a. The upper curve shows the spectrum for the 0.3 AU case, and the lower curve for the 1 AU case. The linear least squares fits to the spectra in the high-frequency ranges ($f > 7 \times 10^{-4}$ Hz for 0.3 AU and $f > 2 \times 10^{-4}$ Hz for 1 AU) are shown by dashed lines. The corresponding fits to the low-frequency ranges (except for a few data points at the lowest-frequency ends) are shown by dotted lines. The spectral index in the high-frequency range is -1.17 ± 0.01 for 0.3 AU and -1.65 ± 0.01 for 1 AU. In the low-frequency range the spectral exponent α equals -0.61 ± 0.03 for 0.3 AU, while $\alpha = -0.35 \pm 0.05$ for 1 AU.

We see that Figures 1a and 1b do not yield exactly the same power law indices. In the low-frequency domain, the derived index shown in Figure 1a is not consistent with the one shown in Figure 1b. Yet, in this spectral range the error bars are comparatively large for both figures. Near the plateau shown by the solid line in Figure 1a, pertaining to the high-frequency part of the spectrum obtained at 0.3 AU, both methods give an index of about -1.2. For the high-frequency range of the spectrum obtained at 1 AU, Figure 1a shows that the index increases from -1.9 to -1.5, which corresponds to the averaged slope of -1.65 as shown in Figure 1b.

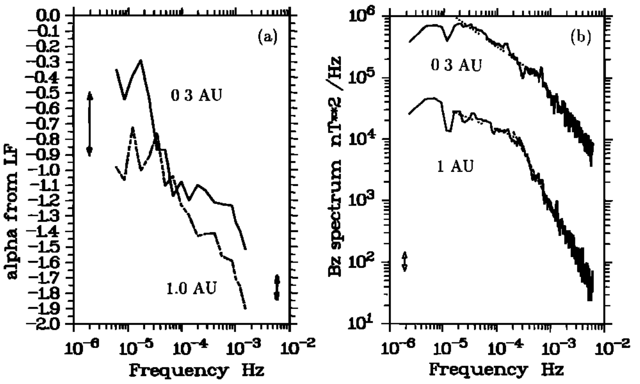

Figure 1. (a) The spectral exponent of B_z calculated from the scaling exponent of the length function through (1). The data were obtained by Helios 2 in 1976 in high-speed streams. The solid line shows the result from data obtained near 0.3 AU, while the dashed line is from near 1 AU observations. For $f > 5 \times 10^{-5}$ Hz the error bar obeys $\Delta\alpha < 0.15$, while for $f < 5 \times 10^{-5}$ Hz one has $\Delta\alpha < 0.4$. (b) Power spectra calculated by using the fast Fourier transform technique with a 31-point-running smoothing procedure (for details, see *Marsch and Tu* [1993a]). The upper curve shows the result for 0.3 AU, while the lower curve gives the result for 1 AU.

We now turn to a discussion of the radial evolution trends. Both Figures 1a and 1b show the same evolution trend. With heliocentric distance increasing from 0.3 to 1 AU, we see that the plateau shown in Figure 1a disappears, which corresponds to the disappearance of the frequency range with a slope near -1.2. The breakpoint frequency, that is, the frequency between the inertial range and the -1 part of the spectrum, moves from the high-frequency end (where it is found at 0.3 AU) to 2×10^{-4} Hz [*Bavassano et al.*, 1982; *Tu et al.*, 1984; *Tu*, 1988; *Roberts*, 1989; *Roberts et al.*, 1990; *Marsch and Tu*, 1990; *Klein*, 1992].

Horbury et al. [1995] presented the spectral index calculated from the length function of magnetic field data obtained by Ulysses in the latitudinal range from 41°S to 51°S and solar distance range from 4.2 to 3.7 AU. As a working hypothesis, let us assume that the original power spectrum of the field fluctuations at 0.3 AU at these high latitudes is similar to the one observed by Helios 2 at 0.3 AU near the equator in a typical high-speed stream, seen at the spacecraft for 13 days. With this assumption we can discuss the radial evolution of the fluctuations by a comparison of the present Helios results and with the Ulysses ones obtained by *Horbury et al.* [1995].

Figure 2 shows the Helios results of Figure 1a (solid line for 0.3 AU and dashed line for 1 AU) plotted against the time lag τ. In Figure 2 we also show the Ulysses results (dotted line) published by *Horbury et al.* [1995] for the spectral exponent α. The difference between the solid and dotted line indicates a clear radial evolution of the slope of the B_z spectrum, while the wind travels from 0.3 to 4 AU, given the validity of the assumption that the Helios exponents are representative of the near-Sun turbulence conditions even out of the ecliptic. This is

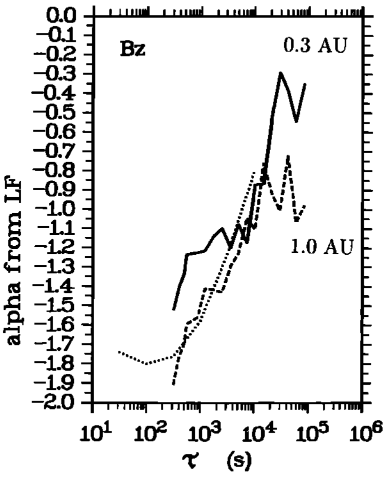

Figure 2. The spectral exponent of B_z calculated from the scaling exponent of the length function against the time delay τ. The solid line is for 0.3 AU, and the dashed line is for 1 AU. The dotted line shows the results that *Horbury et al.* [1995] obtained from Ulysses observations, from 41° to 51° in heliographic latitude and from 4.2 to 3.7 AU in solar distance, for the normal component of the field.

not unreasonable, since the fractional electron density fluctuations decrease with decreasing heliocentric distance (see, e.g., the review by *Tu and Marsch* [1995, and references therein]) and become quite small within $40R_s$, as observed by the Ulysses ranging measurements [*Woo et al.*, 1995]. Therefore close to the Sun the directional fluctuations emanating from coronal holes are, at high as well as low latitudes, widely unaffected by compressive effects and the dynamics of the large-scale streams. The disappearance of the plateau in the solid line corresponds to the disappearance of the frequency range with a -1.2 slope in the spectrum. This is also the major feature of the radial evolution of α at high latitudes, since the dotted line is in fact close to the dashed line in the τ range between 3×10^2 and 10^4 s, corresponding to small-scale magnetic fluctuations.

Horbury et al. [1994] presented the spectrum index of the magnetic fluctuations observed by Ulysses during 160 days in the period when the spacecraft moved from 44°S to 69°S in latitude and from 4.0 to 2.9 AU in solar heliocentric distance. They found that on (spacecraft) timescales of 10^2 to 10^3 s, the spectral index is near 1.7, while on larger scales the spectral index is near 1.1. They concluded that the fluctuations observed by Ulysses are very similar in nature to those observed by the Helios spacecraft in fast solar wind flows around 0.3 AU. However, by a comparison of their result with Figure 1b and the results of *Bavassano et al.* [1982] we found a clear radial evolution. This is a similar spectral evolution as observed for the turbulence near the equator, where the breakpoint frequency is found to move from 6×10^{-3} Hz at 0.3 AU [see *Bavassano et al.*, 1982, Figure 1a] to about 10^{-4} Hz. The observation that at 4 AU at high latitudes the fluctuations have only evolved to a state similar as at 1 AU in the ecliptic indicates, though, that the radial evolution of the high-latitude turbulence is slower. Yet we may safely conclude that a retarded but similar spectral evolution, as compared with the observations of Helios 2 between 0.3 and 1 AU, also takes place in the polar regions, albeit over longer distances.

Conclusions

By a comparison between Helios and Ulysses data (presented by *Horbury et al.* [1994, 1995]), we came to the conclusion that the radial evolution trend in the magnetic spectra, namely that the break-point frequency moves from high to low frequencies according to Helios observations between 0.3 and 1 AU, does also exist in the polar regions of the heliosphere. The underlying assumption is that the Helios observations of high-speed streams at 0.3 AU are representative for the situation prevailing in the polar region of the near-Sun solar wind. If this is true, which is not unreasonable because of the strong reduction of compressive effects in coronal-hole-associated flows near the Sun, then the radial evolution inferred here could fairly well be described by the WKB-like turbulence theory of *Tu et al.* [1984] and *Tu* [1988]. The theory assumes a majority of outward propagating Alfvén waves and a small minority of inward oriented fluctuations. However, presently we do not really know what the origin of the inward oriented fluctuations is, nor do we fully understand the nature of the nonlinear interactions involved. The most probable scenario seems to be nonlinear interactions between outward propagating Alfvén waves and structures, as it was proposed by *Tu and Marsch* [1991, 1993, 1995], *Marsch and Tu* [1993a, b], and *Bruno* [1992]. Whether this idea also applies to the high-latitude fluctuations observed by Ulysses remains to be shown and corroborated in dedicated future data analysis.

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E. Marsch and C.-Y. Tu, Max-Planck-Institut für Aeronomie, D-37189 Katlenburg-Lindau, Germany.

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