Compressible MHD Turbulence: A Source of Heating in the Fast Solar Wind

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Anomalous temperature profile of the solar wind

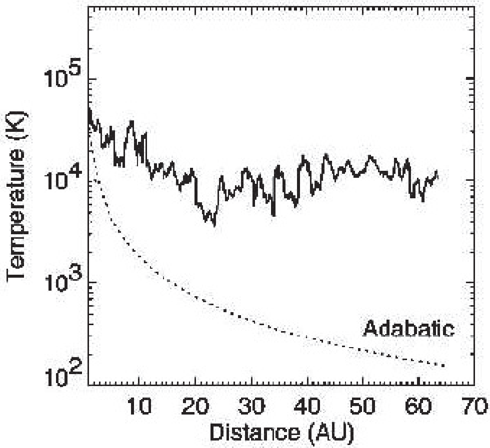


Figure : T(r) of the solar wind (Voyager data, Richardson & Smith, GRL, 2003).

Required heating energy for the fast solar wind

- Power law for temperature profile of the FSW with distances $\longrightarrow T(r) \sim r^{-\xi}$
- Model relating ξ and heating energy flux rate ε_h (Vasquez *et al.*, JGR, 2007)

$$\boxed{\varepsilon_h = \frac{3}{2}\left(\frac{4}{3} - \xi\right)\frac{V_{sw} k_B T(r)}{m_p r},}$$

where V_{SW} be the solar wind speed, k_B the Boltzmann constant, T the temperature, m_P the protonic mass and r is the radial distance of the spacecraft from sun.

- At 1 AU, for $\xi = 0.72$ (from Voyager data) $\Rightarrow$ required heating energy flux rate per unit volume as 5×10^{-17} J.m^{-3}.s^{-1}

Turbulence as a heating source

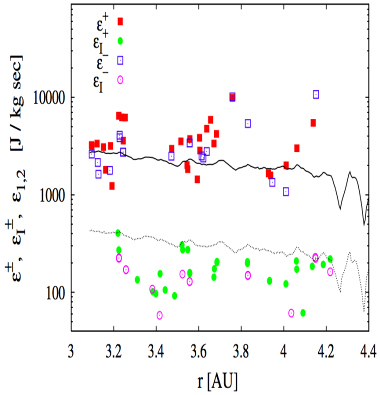


Figure : Turbulent contribution to SW heating (Carbone et al., PRL, 2009)

Incompressible MHD turbulence (Politano & Pouquet, PRE, 1998)

$$\boxed{\left\langle \left(\delta \mathbf{Z}^{\pm}\right)^2 \delta Z_r{}^{\mp} \right\rangle = \frac{4}{3}\varepsilon_I^{\pm} r},$$

where $\mathbf{Z}^{\pm} = \left(\mathbf{v} \pm \frac{\mathbf{b}}{\sqrt{\mu_0 \rho}}\right)$

Heuristic compressible model (Carbone et al., PRL, 2009)

$$\boxed{\left\langle \left(\delta \mathbf{W}^{\pm}\right)^2 \delta W_r{}^{\mp} \right\rangle = \frac{4}{3} \langle \rho \rangle \, \varepsilon^{\pm} r},$$

where $\mathbf{W}^{\pm} = \rho^{1/3}\left(\mathbf{v} \pm \frac{\mathbf{b}}{\sqrt{\mu_0 \rho}}\right)$

However...

- The previous law is heuristic and in this case the model does not carry significant physical meaning.
- The frequency range chosen does not exactly correspond to usual MHD range ($10^{-4} - 10^{-1}$ Hz.).
- No intervals are found for which both of $\varepsilon^{\pm}$ are conserved simultaneously and in fact
- In compressible MHD turbulence, the Pseudo energies $\frac{1}{2}\mathbf{W}^{\pm} \cdot \mathbf{W}^{\pm}$ are no longer inviscid invariants.

Can we do better ?

Exact relation for space plasma turbulence

Isothermal MHD turbulence (Banerjee & Galtier, PRE, 2013)

$$-2\varepsilon = \frac{1}{2}\nabla_{\ell}\cdot \overbrace{\left\langle \left[\frac{1}{2}\delta(\rho\mathbf{z}^-)\cdot\delta\mathbf{z}^- + \delta\rho\delta e\right]\delta\mathbf{z}^+ + \left[\frac{1}{2}\delta(\rho\mathbf{z}^+)\cdot\delta\mathbf{z}^+ + \delta\rho\delta e\right]\delta\mathbf{z}^- + \bar{\delta}(e+\frac{v_A^2}{2})\delta(\rho\mathbf{z}^- + \rho\mathbf{z}^+)\right\rangle}^{\text{Usual flux term}}$$

$$\underbrace{-\frac{1}{4}\left\langle \frac{1}{\beta'}\nabla'\cdot(\rho\mathbf{z}^+e') + \frac{1}{\beta}\nabla\cdot(\rho'\mathbf{z}'^+e) + \frac{1}{\beta'}\nabla'\cdot(\rho\mathbf{z}^-e') + \frac{1}{\beta}\nabla\cdot(\rho'\mathbf{z}'^-e)\right\rangle}_{\text{New type of flux term}}$$

$$+\left\langle(\nabla\cdot\mathbf{v})\left[R'_E - E' - \frac{\bar{\delta}\rho}{2}(\mathbf{v_A}'\cdot\mathbf{v_A}) - \frac{P'}{2} + \frac{P'_M}{2}\right]\right\rangle + \left\langle(\nabla'\cdot\mathbf{v}')\left[R_E - E - \frac{\bar{\delta}\rho}{2}(\mathbf{v_A}\cdot\mathbf{v_A}') - \frac{P}{2} + \frac{P_M}{2}\right]\right\rangle$$

$$+\langle(\nabla\cdot\mathbf{v_A})\left[R_H - R'_H + H' - \bar{\delta}\rho(\mathbf{v}'\cdot\mathbf{v_A})\right]\rangle + \langle(\nabla'\cdot\mathbf{v_A}')\left[R'_H - R_H + H - \bar{\delta}\rho(\mathbf{v}\cdot\mathbf{v_A}')\right]\rangle$$

where

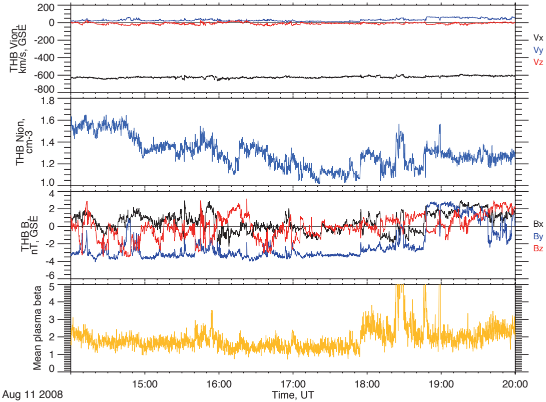
$$E = \rho(\mathbf{v}\cdot\mathbf{v} + \mathbf{v}_A\cdot\mathbf{v}_A)/2 + \rho e,\ \ E' = \rho'(\mathbf{v}'\cdot\mathbf{v}' + \mathbf{v}'_A\cdot\mathbf{v}'_A)/2 + \rho' e';$$

$$R_E = \rho(\mathbf{v}\cdot\mathbf{v}' + \mathbf{v}_A\cdot\mathbf{v}'_A)/2 + \rho e',\ \ R'_E = (\rho'\mathbf{v}'\cdot\mathbf{v} + \mathbf{v}'_A\cdot\mathbf{v}_A)/2 + \rho' e;$$

$$R_H = \rho(\mathbf{v}\cdot\mathbf{v}'_A + \mathbf{v}'\cdot\mathbf{v}_A)/2,\ \ R'_H = \rho'(\mathbf{v}'\cdot\mathbf{v}_A + \mathbf{v}\cdot\mathbf{v}'_A)/2$$

$$H = \rho\mathbf{v}\cdot\mathbf{v}_A,\ \ H' = \rho'\mathbf{v}'\cdot\mathbf{v}'_A;\ \ \beta = 2C_S^2/v_A^2;\ \beta' = 2{C_S'}^2/{v_A'}^2$$

Exploitation of compressible MHD turbulence model in solar wind



Solar wind data source: Themis B (NASA)

Plasma data : Electrostatic analyzer (ESA) – resolution 3 s.

magnetic data: Fluxgate Magnetometer (FGM) – resolution 3 s.

Compressible scaling in the solar wind

Schematic expression of newly derived exact relation

$$-2\varepsilon = \frac{1}{2}\nabla_{\mathbf{r}} \cdot \langle \mathcal{F}_1 + \mathcal{F}_2 + \mathcal{F}_3 \rangle + \langle \Phi \rangle + \text{source terms}$$

where

$$\langle \mathcal{F}_1 \rangle = \left\langle \left[\frac{1}{2}\delta(\rho \mathbf{z}^-) \cdot \delta \mathbf{z}^-\right] \delta \mathbf{z}^+ + \left[\frac{1}{2}\delta(\rho \mathbf{z}^+) \cdot \delta \mathbf{z}^+\right] \delta \mathbf{z}^- \right\rangle,$$

$$\langle \mathcal{F}_2 \rangle = \langle 2\delta\rho\delta e\delta \mathbf{v} \rangle,$$

$$\langle \mathcal{F}_3 \rangle = \left\langle 2\bar{\delta}(e + \frac{v_A^2}{2})\delta(\rho \mathbf{v}) \right\rangle,$$

$$\langle \Phi \rangle = -\frac{1}{2}\left\langle \frac{1}{\beta'}\nabla' \cdot (\rho \mathbf{v} e') + \frac{1}{\beta}\nabla \cdot (\rho' \mathbf{v}' e) \right\rangle.$$

Intervals of uniform plasma beta

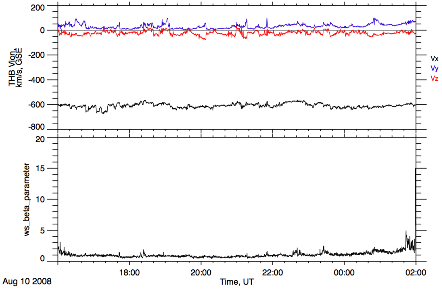


Figure : 10 hours interval with uniform β ($\simeq 1$)

For the intervals with $\beta \simeq 1$

$$\langle \Phi \rangle \approx -\frac{1}{2} \nabla_{\mathbf{r}} \cdot \left\langle \rho \mathbf{v} e' - \rho' \mathbf{v}' e \right\rangle = \nabla_{\mathbf{r}} \cdot \left\langle \overline{\delta} e \delta(\rho \mathbf{v}) \right\rangle$$

Heating energy from compressible MHD turbulence

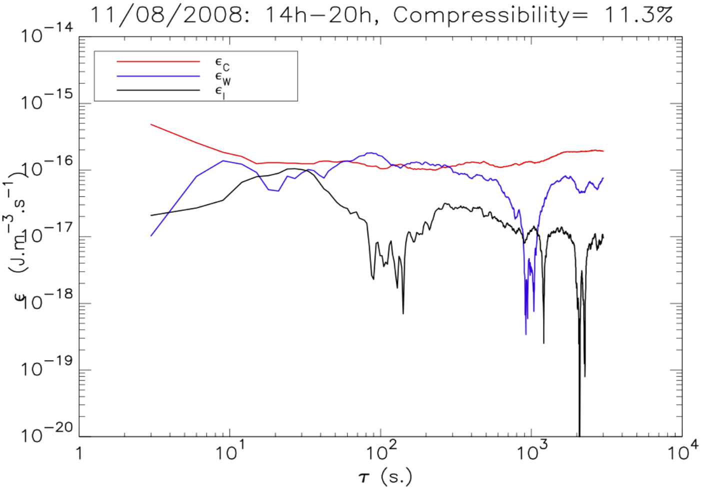


Figure : Estimates of energy cascade rate of different models

Comparison of different terms

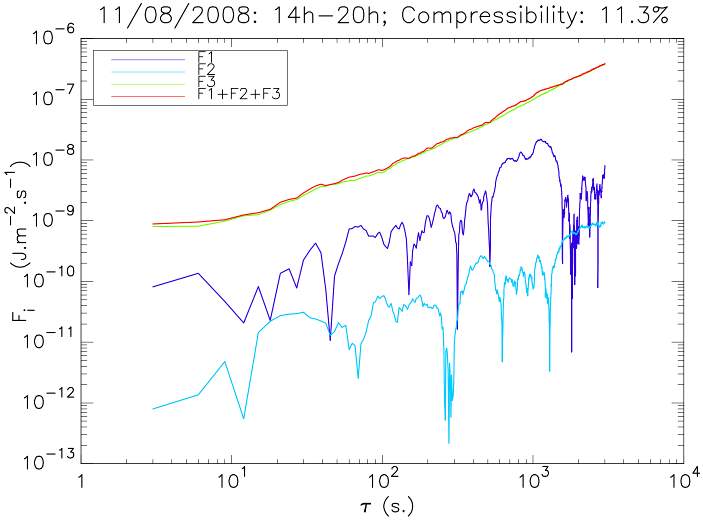


Figure : Comparison of the different terms F_1, F_2 and F_3.

Thank You